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FRACTIONAL ORDER ADAPTIVE GENERALIZED PREDICTIVE CONTROL DESIGN BASED ON ROMERO GPC OPTIMIZATION CRITERION

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Abstract

In this paper, we propose a new fractional-order adaptive generalized predictive control (FA-GPC) design based on a fractional order Romero GPC cost function and online estimation of the plant model using the Recursive Least Square (RLS) algorithm. The plant model is supposed to be linear time-invariant with unknown parameters. The proposed controller is applied in numerical simulation to a DC motor system and compared with the classical adaptive generalized predictive control (A-GPC). Simulation results illustrate the effectiveness of the proposed adaptive FA-GPC controller.

1. Introduction

Since more than three hundred years there has been a wealth of literature on fractional calculus [1]. Nowadays, it has emerged as one of the dominant research field with applications in a variety of technology and science disciplines. It has been recognized that the introduction of fractional order calculus in control systems enhances robustness and performance [2, 3]. Analysis and design approaches based on fractional calculus have been proposed to extend classical control theory. As examples of fractional order controller, FOC, we can cite the Fractional $PI^{\lambda}D^{\mu}$ controller [4], the CRONE controller [5], and the fractional adaptive controller [6].

Generalized predictive controller GPC is a model based receding horizon control algorithm. At each sampling period, it minimizes a multistage cost function in order to find a future sequence of controls on a prediction horizon and only the first element of the sequence is applied to the system [7]. GPC is recognized as one of the few controllers that has been applied successfully in many industrial processes [8, 9]. This is due to its ease of implementation and tuning.

Whenever the model of the system is known, GPC results in a linear control law that is easy to implement. However, if the model is unknown or varying in time, the control law has to be update in real time. This is usually carried out using a self tuning algorithm. This is possible since many industrial processes can be represented by Ziegler and Nichols type transfer functions [10, 11]. This task remains very difficult, and only few adaptive MPC algorithms have reached the industry marketplace. More research work is needed in order to understand and respond to practical applications requirements and challenges [12-14].

Recently, fractionals order controllers and control of fractional order system have attracted considerable interest. In this context, a variety of approaches have been proposed [15-17]. Considerable interest has been focused on model predictive control for fractional order systems or/and involving fractional order control laws [18, 19]. Various control schemes and applications of this concept were proposed in literature like MPC with fractional order models [20-22], fuzzy predictive control of fractional-order nonlinear discrete-time systems [23], GPC control applied for a fractional order dynamic model of solid oxide fuel cell output power [24], fractional-order GPC applied for low-speed control of gasoline-propelled cars [25] ...etc.

In this work, we propose a new fractional order adaptive GPC control scheme based on the configuration of the fixed Fractional-order Generalized Predictive Control (FGPC) designed by Romero et al. [26, 27]. It is a generalization of GPC when we use the fractional order integral operator in the cost function. Unfortunately, such fixed control scheme becomes vulnerable when the process parameters are unknown or varying in time [28].

The main contribution of this paper is to propose a new adaptive control algorithm based on GPC control scheme and a prediction cost function involving fractional order integrators. The process model parameters will be identified in real time using the least square estimation method (RLS).

This paper is structured as follows: In Section 2, some preliminaries on fractional order operators are presented. Section 3 gives a brief introduction to GPC and a thorough mathematical formulation of FGPC is carried out. In Section 4, the proposed fractional-order adaptive GPC

(FAGPC) is developed. An application example is proposed in Section 5 with a comparative study with classical AGPC. Finally, Section 6 draws the main conclusions of this work.

2. Fractional Order Systems

Non-integer integrals and derivatives have been known since the development of regular calculus. The Riemann-Liouville (R-L) and Grünwald-Letnikov (G-L) definitions are the commonly used definitions of the fractional order integrals and derivative [29].

The R-L integral of real order $\eta > 0$ is defined as

$$I_{RL}^{\eta} h(t) = D_{RL}^{-\eta} h(t) = \frac{1}{\Gamma(\eta)} \int_0^t (t-\tau)^{\eta-1} h(\tau) d\tau, \tag{1}$$

and derivative of fractional order $\eta > 0$ is given by

$$\begin{aligned} {}_{RL}D^{\eta} h(t) &= \frac{d}{dt}\left[D_0^{-(1-\eta)} h(t)\right] \\ &= \frac{1}{\Gamma(1-\eta)} \frac{d}{dt} \int_0^t (t-\tau)^{-\eta} h(\tau) d\tau, \end{aligned} \tag{2}$$

with the Gamma function given by

$$\Gamma(x) = \int_0^{\infty} y^{x-1} e^{-y} dy, \tag{3}$$

such that $(a, t) \in \Re^2$ with $a < t$ and $(0 < \eta < 1, \eta \in \Re)$.

The Grünwald-Letnikov definition is expresses as

$${}_{GL}D^{\eta} g(t) = \lim_{h \to 0} h^{-\eta} \sum_{r=0}^{\left[\frac{t-a}{h}\right]} (-1)^r \frac{\Gamma(\eta+1)}{r!\,\Gamma(\eta-r+1)} g(t-rh), \tag{4}$$

where h is the time increment, $\left[\frac{t-a}{h}\right]$ is a flooring operator and the binomial coefficients (for a positive integer r) are given by

$$\binom{\eta}{0} = 1, \binom{\eta}{r} = \frac{\eta(\eta-1)\ldots(\eta-r+1)}{r!}. \tag{5}$$

3. Classical Adaptive GPC Scheme

In many practical situations, the model of the system may be unknown or partially known. For this reason we need to implement an adaptive algorithm. In this work, we use the so-called recursive least squares method formulated by Gauss [30], with GPC control and we have the adaptive GPC.

3.1. GPC control algorithm

We consider the CARIMA (Controlled Auto-Regressive Integrated Moving-Average) model given as follows [31]:

$$A(z^{-1})y(k) = B(z^{-1})u(k-1) + C(z^{-1})\xi(k)/\Delta, \tag{6}$$

where u is control input, y is output and ξ is an uncorrelated random sequence. Δ is the difference operator $1-z^{-1}$; $C(z^{-1})$ is chosen to be 1. The polynomials A and B are given as

$$A(z^{-1}) = 1 + \sum_{i=1}^{n_a} a_i z^{-i}, \tag{7}$$

$$B(z^{-1}) = \sum_{i=1}^{n_b} b_i z^{-i}. \tag{8}$$

A cost function is the basis of the GPC algorithm, is of the form:

$$J = \sum_{j=N_1}^{P} \gamma_j [y(k+j) - y_r(k+j)]^2 + \sum_{j=1}^{M} \lambda_j [\Delta u(k+j-1)]^2, \tag{9}$$

where

- P and N_1 are the maximum and minimum predictive step respectively;
- M and λ_j are the maximum control step and a control weighting sequence respectively;
- $y_r(k+j)$ is a reference sequence.

The cost function (9) can be rearranged as

$$J = e^T \gamma e + \Delta u^T \lambda \Delta u, \tag{10}$$

where

$$\gamma = \mathrm{diag}(\gamma_1, \dots, \gamma_M), \tag{11}$$

$$\lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_M). \tag{12}$$

Minimizing J (with the constraint equal to zero after M samples) gives the incremental control vector

$$\Delta U = (G^T \gamma G + \lambda)^{-1} G^T (y_r - f_y) \tag{13}$$

G is a $(p - N_1 + 1)M$ matrix with the step response coefficients of the model. f_y is the part of the future output that depends on the future control Δu. We have

$$y_r = [y_r(k+1), \dots, y_r(k+P)]^T,$$

$$\Delta U(k) = [\Delta u(k), \dots, \Delta u(k+M-1)]^T. \tag{14}$$

The current control $u(k)$ is given by

$$u(k) = u(k-1) + \Delta u(k), \tag{15}$$

where $\Delta u(k)$ is the first element of $\Delta U(k)$.

3.2. Estimation

Many recursive estimation methods may be used to estimate the coefficients of the polynomials A and B. In this work we will use the Recursive Least-Squares (RLS) estimator [32]. It is a basic technique for parameter estimation. The method is particularly simple if the model has the property of being linear [33, 34].

Let the process model be written as follows:

$$\begin{aligned} y(t) = & -a_1 y(t-1) - a_2 y(t-2) - \ldots - a_n y(t-n) \\ & + b_0 u(t-d_0) + b_1 u(t-d_0-1) + \ldots + b_m u(t-d_0-m). \end{aligned} \tag{16}$$

It can be written as

$$y(t) = \phi^T(t-1)\theta, \tag{17}$$

where

$$\begin{aligned} \theta^T &= [a_1 \;\; a_2 \ldots a_n \;\; b_0 \;\; b_1 \ldots b_m], \\ \phi &= [-y(t-1) - y(t-2) \ldots - y(t-n)\, u(t-d_0)\, u(t-d_0-1) \\ & \qquad \ldots u(t-d_0-m)]. \end{aligned} \tag{18}$$

The recursive least-squares estimator with exponential forgetting is given by

$$\begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t-1) + K(t)\epsilon(t), \\ \epsilon(t) &= y(t) - \phi^T(t-1)\hat{\theta}(t-1), \\ K(t) &= P(t-1)\phi(t-1)\left(\tilde{\lambda} + \phi^T(t-1)P(t-1)\phi(t-1)\right)^{-1}, \\ P(t) &= I - K(t)\phi(t-1)P(t-1)/\tilde{\lambda}. \end{aligned} \tag{19}$$

If the input signal is exciting enough and the estimated model structure is compatible with the process, the estimates will converge to their true values [28].

4. Fractional Order Adaptive Generalized Predictive Control

Fractional calculus is the mathematical analysis whose objective is to investigate and apply integrals and derivatives of arbitrary order.

Recently, it has been widely applied in automatic control engineering for designing new efficient, powerful and robust control schemes, particularly in fractional adaptive control [35, 36].

Consider the fractional-order definite integration operator

$$^{\alpha}I_{a}^{b}, \tag{20}$$

where α is the fractional order, a and b are the integral bounds.

Introducing this operator in a cost function of GPC makes a fractional order GPC. It is a generalization of GPC based of CARIMA model. The fractional order adaptive GPC controller configuration is represented in Figure 1.

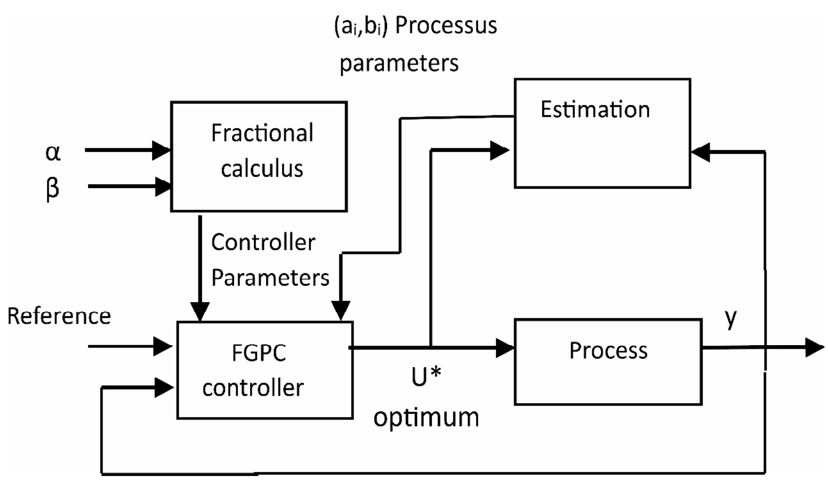

Figure 1. Block diagram for fractional order adaptive GPC controller.

The cost function of the FGPC has the following expression:

$$J_F = {}^{\alpha}I_{N_1}^{P}[e^2(t)] + {}^{\beta}I_{N_1}^{M}[\Delta u^2(t-1)], \tag{21}$$

$$J_F = \int_{N_1}^{P} D^{1-\alpha}[e^2(t)]dt + \int_{N_1}^{M} D^{1-\beta}[\Delta u^2(t-1)]dt, \tag{22}$$

where α and β are positive real numbers.

The integration operator ${}^{\alpha}I_a^b$ in Equation (20) can be discretized with a sampling period Δt as

$${}^{\alpha}I_a^b f(t) = \int_a^b [D^{1-\alpha} f(x)]dx = \Delta x^{\alpha} W^T F, \tag{23}$$

where

$$W = (\dots\ w_b\ \ w_{b-1} \dots w_{n+1} \dots w_0)^T, \tag{24}$$

$$F = (\dots\ f(0)\ f(\Delta x)\ \dots f(a-\Delta x)\ f(a)\ \dots f(b-\Delta x)\ f(b))^T. \tag{25}$$

After discretization of J_F we obtain

$$J_F = \Delta t^{\alpha} e^T \hat{\gamma} e + \Delta t^{\beta} \Delta u^T \hat{\lambda} \Delta u. \tag{26}$$

With $\hat{\gamma}$ and $\hat{\lambda}$ infinite-dimensional square weighting matrices. Thus, the latter expression has infinite memory and J_F depends both on an infinite number of past terms of e and Δu and a finite number of predicted future terms. To take into account just the future values in the intervals of interest, P and M, we shall truncate (26) as

$$J_F = e^T \gamma(\alpha,\ \Delta t) e + \Delta u^T \lambda(\beta,\ \Delta t) \Delta u, \tag{27}$$

where $\gamma(\alpha, \Delta t)$ and $\lambda(\beta, \Delta t)$ are given by Equations (28) and (29), respectively. By doing so we obtain a classical GPC formulation with weighting sequences γ and λ that are given by the sampling time Δt and the fractional orders of derivation α and β.

$$\gamma(\alpha, \Delta t) = \Delta t^{\alpha} \text{diag}(w_{P-1} \ldots w_1 \ w_0), \tag{28}$$

$$\lambda(\beta, \Delta t) = \Delta t^{\beta} \text{diag}(w_{M-1} \ldots w_1 \ w_0). \tag{29}$$

The optimal control law is obtained by minimizing (26) using conventional GPC techniques [37].

$$u^{\star} = (G^T \gamma_{\rightarrow} G + \lambda_{\rightarrow})^{-1} G^T \gamma_{\rightarrow} E_0, \tag{30}$$

where the symbol $_{\rightarrow}$ represents the future values and

$$E_0 = (e_0(t+1) \ldots e_0(t+P)). \tag{31}$$

The most important element in GPC is the model of the process/plant, which is identified here by the RLS method (its parameters). By combining the RLS estimation of Equations (6) with the FOGPC, we obtain the fractional order adaptive GPC regulator represented in the following algorithm:

Algorithm: (Fractional order adaptive GPC)

Specifications: Parameters $N_1, P, M, T_e, \alpha, \beta$ and the initial vector of ϕ_i.

Step 1: Calculate the matrices γ and λ from the Equations (28) and (29).

Step 2: Estimate the coefficients of polynomials A and B in Equation (6) using RLS method.

Step 3: Calculate the matrix G with the step response coefficients of the novel coefficient (Ai, Bi) of model.

Step 4: Calculate u optimum given by Equation (30).

Step 5: Compute the output.

Step 6: Go to **Step 2**.

5. Simulation Results

In order to illustrate the robustness proprieties of the proposed FA-GPC, an example comparing the FA-GPC and the adaptive GPC is presented. It corresponds to a direct current motor velocity [38]. This system represents a typical industrial process and because of this, it is normally used to evaluate the performance of industrial controllers.

5.1. Model of DC motor

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translation motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in Figure 2.

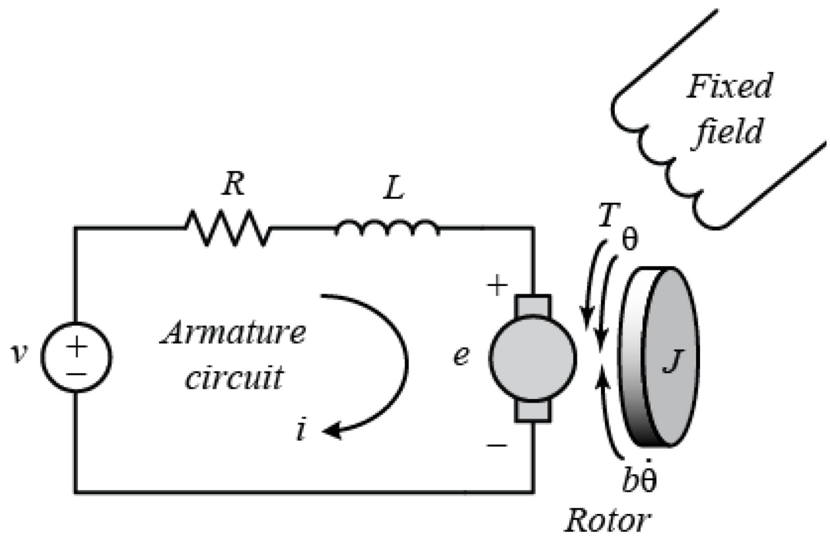

Figure 2. The structure of a DC motor.

For this example, we will assume that the input of the system is the voltage source (V) applied to the motor's armature, while the output is the rotational speed of the shaft $d\theta/dt$. The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

The physical parameters for this example are presented in Table 1.

Table 1. The DC motor physical parameters

Specification Parameter	Value
Moment of inertia of the rotor (J)	0.018kg.m^2
Motor viscous friction constant (b)	0.0055N.m.s
Electromotive force constant (K_e)	1V/rad/sec
Motor torque constant (K_t)	0.01N.m/Amp
Electric resistance (R)	6.25Ω
Electric inductance (L)	0:024H

From the scheme of Figure 2, we can derive the following governing equations based on Newton's 2nd law and Kirchhoff's voltage law. Applying the Laplace transform, we arrive at the following open-loop transfer function by eliminating $I(s)$ between the two above equations, where the rotational speed is considered as the output and the armature voltage is considered as the input.

By replacing the parameters' values from Table 1, we get

$$G(s) = \frac{81018}{s^2 + 260.7s + 2394}. \tag{32}$$

The discrete time model with the sampling period $T = 0.04s$ is given by

$$G_d(z) = \frac{9.816z + 0.9112}{z^2 - 0.683z + 2.959\,10^{-5}}. \tag{33}$$

5.2. Results and discussion

In this simulation, we consider a second order model of DC motor given by the transfer function:

$$G_d^e(z) = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2}, \tag{34}$$

where b_0, b_1, a_1, and a_2 are obtained from the RLS estimator.

Simulation are performed using the initial values $\theta_0 = [5111]$, $\tilde{\lambda} = 0.95$.

Besides, the adaptive AGPC and fractional order FAGPC controllers will be calculated using (34) as the process model and settings: $N_1 = 1$, $P = 18$, and $M = 3$.

The AGPC controller is tuned with constant sequences $\lambda = 10^3$, $\gamma = 1$.

For the proposed FAGPC controller, the weighting sequences λ and γ are determined from the corresponding fractional-differentiation orders α and β, using Equations (28) and (29).

Two different settings of this controller are used in simulations:

- The controller FAGPC_1 is tuned with $\beta = 0.6$ and $\alpha = 2.9$.
- The controller FAGPC_2 is tuned with $\beta = 0.1$ and $\alpha = 1.9$.

For these controllers, the weighting sequences $\gamma(\alpha, \Delta t)$ and $\lambda(\beta, \Delta t)$ are respectively as follows:

$$\begin{cases} \gamma(2.9,\ 0.04) = \mathrm{diag}(0.011\ \ 0.0099\ \ 4.99\,10^{-4}\ \ 2.5\,10^{-4}\ \ 8.83\,10^{-5}), \\ \lambda(0.6,\ 0.04) = \mathrm{diag}(0.6784\ \ 0.4204\ \ 0.1450). \end{cases} \tag{35}$$

$$\begin{cases} \gamma(1.9,\ 0.04) = \mathrm{diag}(0.0287\ \ 0.0293\ \ 0.0061\ \ 0.0042\ \ 0.0022), \\ \lambda(0.1,\ 0.04) = \mathrm{diag}(1.272\ \ \ 1.3771\ \ \ 0.7248). \end{cases} \tag{36}$$

Simulation results are represented in Figure 3 to Figure 7.

From Figure 3, it is obvious that the proposed fractional order adaptive GPC approach is able to improve the dynamical behaviour of the adaptive GPC regulator, with more rapidity of convergence and less static error.

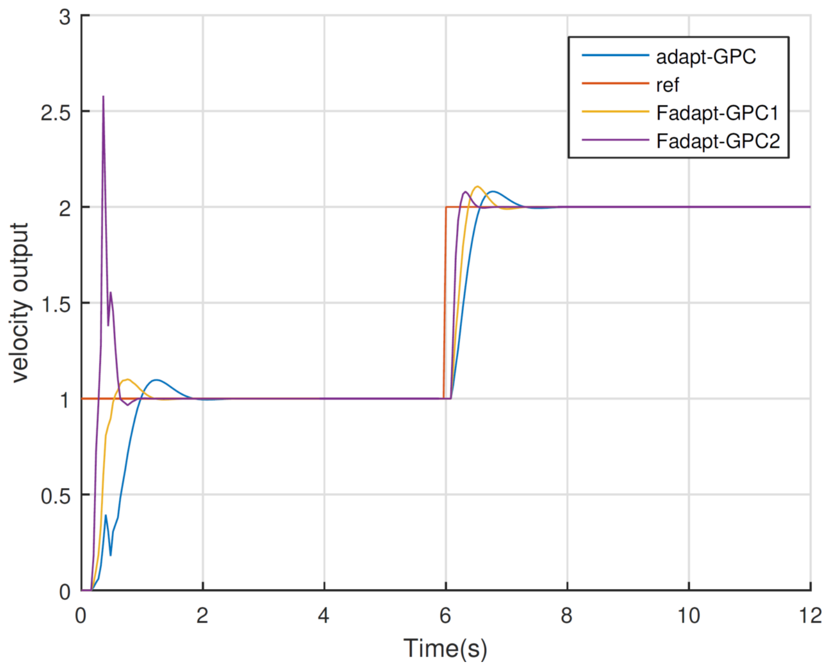

Figure 3. Comparative output responses.

We remark that the system follows the reference signal with hard overshoot in the case of FA-GPC_2 but it is eliminated in the case of FA-GPC_1 because, each pair (α, β) defines a different FGPC controller.

Figure 4 shows that this performance improvement needs more energetic input control signals, which are relatively low in case of FA-GPC_1.

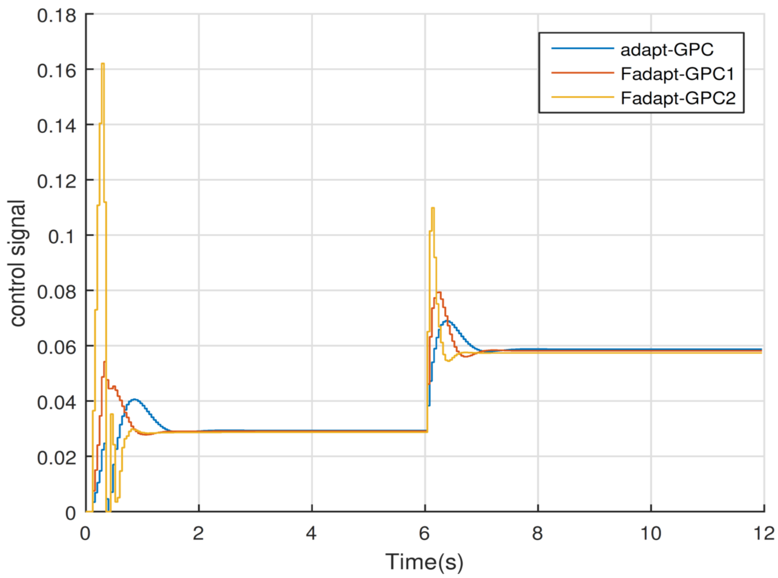

Figure 4. Control signals.

Figure 5 to Figure 7 illustrate the rapid convergence of the parameters estimation algorithm for all the control configurations.

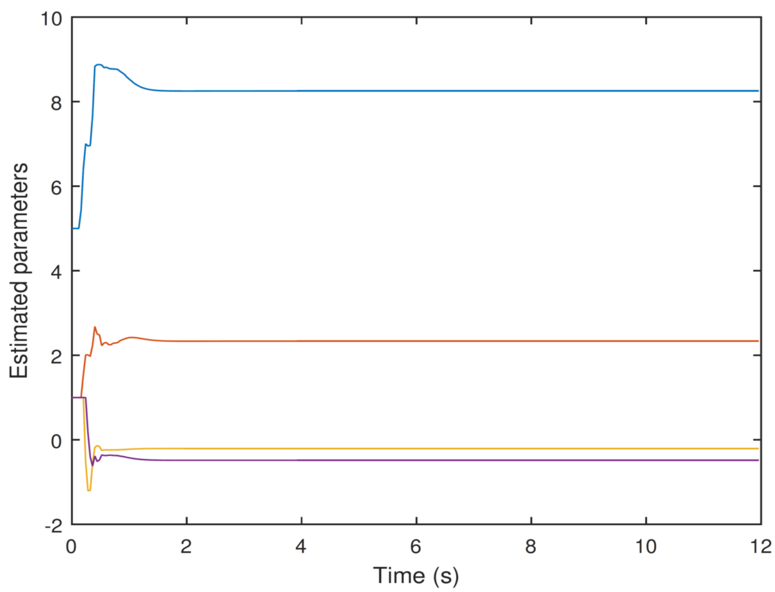

Figure 5. Estimated parameters obtained by adaptive GPC.

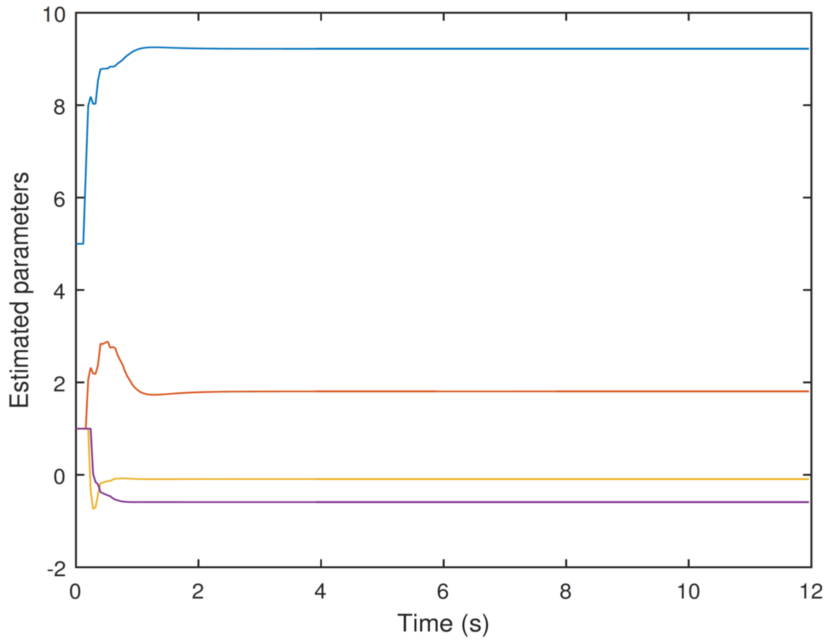

Figure 6. Estimated parameters obtained by fractional adaptive GPC_1.

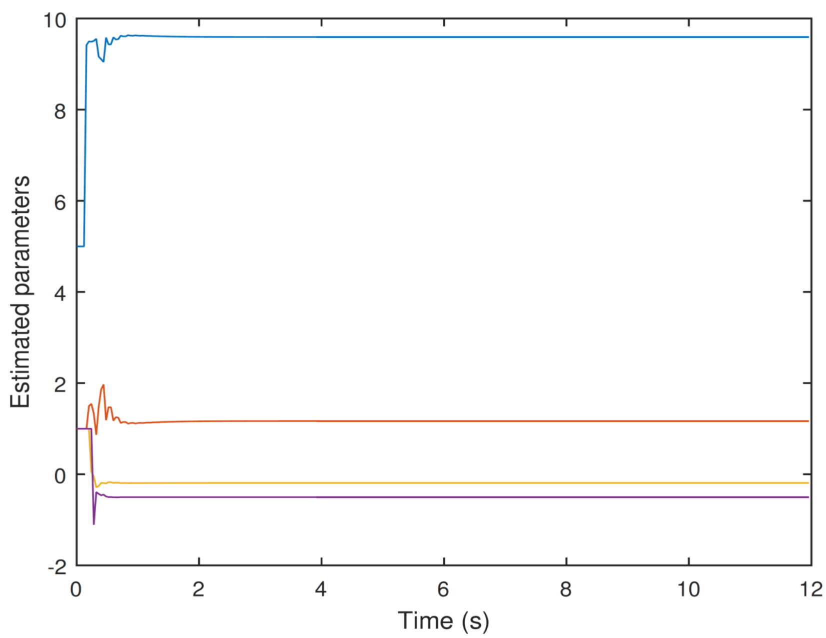

Figure 7. Estimated parameters obtained by fractional adaptive GPC_2.

6. Conclusion

We propose a new design of fractional-order adaptive generalized predictive control with an application to the control of a DC motor. The control configuration is a generalization of the Romero's fixed FGPC. An RLS algorithm is used for the controller parameters' updating.

Simulation results on the DC motor are compared to the ones obtained using a classical adaptive GPC strategy. They illustrate the effectiveness of the proposed fractional adaptive control scheme. Moreover, different weighting sequences in the cost function J can be easily defined using just two parameters, the fractional orders α and β.

Further researches will concern the application this approach to plants with delays, and investigate the FA-GPC robustness enhancement.

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